SEMIOTIC POTENTIAL OF INQUIRING-GAME ACTIVITIES

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We integrate the Theory of Semiotic Mediation with a phenomenological perspective to analyse the semiotic potential of GeoGebra when inquiring-game activities are implemented. These activities aim at developing a theoretical approach towards geometry in secondary school students, starting from game situations. An inquiry-game activity based on a property of isosceles triangle is presented and experimental data from a grade 9 classroom are analysed. Results show how the inquiring-game activity contributes to the production of artefact signs that can evolve into mathematical meanings, and of different layers of meaning developed within the contexts evoked by these signs.

INTRODUCTION AND THEORETICAL FRAMEWORK

Research has shown that the use of Dynamic Geometry Environments (DGEs) can contribute positively in students' harmonization of theoretical and empirical aspects of geometry (Laborde et al., 2005). In particular, dragging tools have been proved to allow students experiencing the variation hinted in static diagrams, and so improving their ways of seeing and conceiving them as different instances of the same geometrical objects (Leung, 2003), beyond the prototypical cases that are usually considered in classical teaching (e.g. a rectangular with a longer horizontal side and a shorter vertical side). Since the creation of the first DGEs, researchers and teachers devoted their attention to *robust diagrams* (Healy, 2000), i.e. diagrams constructed according to geometrical properties of the figures. In these diagrams, the geometric properties of the figure remain invariant when its base-points are dragged. On the contrary, in *soft diagrams* properties are empirically constructed: hence dragging cannot be used for testing the construction and discerning the invariants (*ibid.*).

In our study we focus on *soft diagrams* within *inquiring-game activities* aimed at introducing students to the theoretical aspects of geometry (e.g. distinguishing between hypothesis and thesis in a theorem, reasoning on properties, formulating conjectures, etc.). Inquiring-game activities (Soldano & Arzarello, 2018) are inspired by games of verification and falsification, known as *semantical games*, used by the logician J. Hintikka (1998) to establish the truth of statements in his Game Theoretical Semantic. According to this theory, in order to establish the truth of statements expressed in the form $\forall x\ \exists y \mid S(x,y)$, we can imagine a game in which a player, called Falsifier (F) controls variable x and has the goal to show that the statement is false and another player, called Verifier (V) controls variable y and has to show its truth. In order to win F will look for the worst values of variable x. If, even in the worst-case scenarios, the verifier is able to find a suitable value of y, then the statement is true; otherwise it is

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false. Within a DGE we conceive x and y as base-points of a *soft diagram* and $S(x, y)$ as a geometric property (e.g. being an isosceles triangle). During the game, F typically tries to create troubles for the verifier by proposing non-prototypical diagrams. V, on the contrary, will try to show the possibility to achieve his goal in all situations, also in the non-prototypical ones.

Although each game is designed on a geometric theorem, it is not granted that this foreseen mathematical meaning does emerge when the students play the game. This aspect is framed within the Theory of Semiotic Mediation (TSM) with the notion of *semiotic potential of an artefact*: it refers to the double semiotic relationship occurring on one side, between the artefact and the personal meanings elaborated by the students when they use it, and on the other side, between the artefact and the mathematical meanings foreseen by an expert (Bartolini Bussi & Mariotti, 2008). Through the analysis of signs produced by students and teacher, it is possible to document how this double relationship emerges and evolves. The basic signs for the development of the semiotic potential are produced when the students use the artefact and are called *artefact signs;* also thanks to the teacher's intervention these signs can evolve, first into *pivot signs* relating both to the specific instrumented actions and to the mathematical domain, and then into *mathematical signs* that refer to the mathematical domain as culturally and historically developed.

Also phenomenological perspectives on mathematical teaching-learning processes points out how a delicate process is to guide students to "see and recognize things according to 'efficient' cultural means" and to convert their "eye (and other human senses) into a sophisticated intellectual organ" (Radford, 2010, p. 4). As Radford stresses, it is necessary to promote a "lengthy process of domestication" (*ibid.*) of the way they are looking at things while learning mathematics. Depending on the ages and the backgrounds of the students, a given mathematical situation may evoke different contexts and lead to different sense-making (e.g. an increasing and decreasing continuous graph in a Cartesian plane may be seen as the picture of a mountain by young students). According to the mathematician and philosopher G. C. Rota (1991), the starting point for understanding how mathematical meaning arise and evolve is the key assumption that there is "no such thing as true seeing", but "there is only *seeing as*" (*ibid*., p. 239). In Rota's account, this process is referred to as *disclosure*. Disclosure is a Husserlian concept that indicates the process by which people make sense of and interpret the various situations of the world in the contexts in which they are exposed. Different contexts may provide different meanings to the objects of our knowledge; furthermore, these different contexts are not isolated, on the contrary they are *layered* upon one another, and the layers can generate *different (layered) meanings* in the flow of time.

Adopting Rota's phenomenological account, Arzarello, Ascari, Baldovino, and Sabena (2011) showed how suitable didactic techniques made by the teacher may promote different layers of meanings in students' disclosures of calculus concepts, highlighting the role of *making present* things that are absent, and of *prompting* students' *attention* on some specific aspects of the contexts (Mason, 2008).

Our study is grounded on the research hypothesis that using *soft diagrams* within the inquiry game can initiate the students' disclosure of the meaning of a geometric theorem because the necessary/sufficient condition for the thesis are not hidden in the construction steps but are produced by intentional moves made on *soft diagrams* within DGEs. Under this hypothesis, in this paper we investigate which artefact/pivot/mathematical signs emerge in the students' disclosure processes based on inquiring-games and how the contexts in which these signs are produced contribute to the development of different layers of meanings.

METODOLOGY

The study grounds on a pilot teaching experiment on geometric inquiring-games carried out in an Italian grade 9 classroom. Twelve students were involved for about two hours in inquiring-games and in answering some guiding questions, and for other two hours in a classroom discussion under the guidance of the teacher. We took part to the experiment as participant observers, in collaboration with the teacher. Collected data consist of the videorecording and screen capture of two pairs of students while playing the game and answering the worksheet questions, the completed worksheets from all the students, and the videorecording of the classroom discussions. Videorecorded students were chosen by the teacher among those who felt self-confident in front of video cameras. Data are firstly analysed according to the lens of TSM, in order to seize the evolution from artefact signs to mathematical signs produced within inquiring-games activities. In order to deepen the interpretation of the evolution of signs, the analysis is then integrated with a phenomenological lens.

We present some results from the first inquiring-game activity, which is played on the following theorem (T_{MB}): "*if the median and the angle bisector drawn from the same vertex of a triangle coincide, then the triangle is isosceles*".

Within your pair, establish a verifier who moves point B and a falsifier who moves point C. Each match is made by two moves and the first one is always made by the falsifier. The verifier's goal is to make segment CD and line b coincide; the falsifier's goal is to prevent the verifier from reaching his/her goal.

The winner of the match is the player who reaches the goal at the end of the verifier's move.

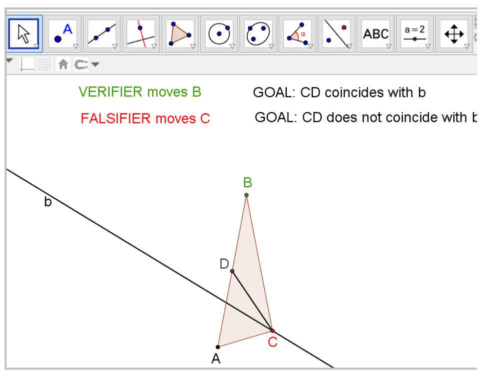

Table 1: Initial configuration and rules of the game (English translation)

The game is played in a GeoGebra file containing a dynamic triangle ABC (A is fixed, while B and C are free points), a segment CD robustly constructed as median from the vertex C and a line b robustly constructed as angle bisector from the vertex C (see Table 1). By moving B and C, the triangle changes and consequently also the positions of the median and of the angle bisector change. However, being *robust diagrams*, they preserve their constructive nature, i.e. they always remain median and angle bisector. On the contrary, isosceles triangles may be obtained as *soft diagrams*. In particular, the falsifier may produce triangles in which the median and the angle bisector do not coincide (non-isosceles triangles, Fig.2a), and the verifier may produce isosceles triangles in which the median and the angle bisector coincide to the eye (Fig.2b).

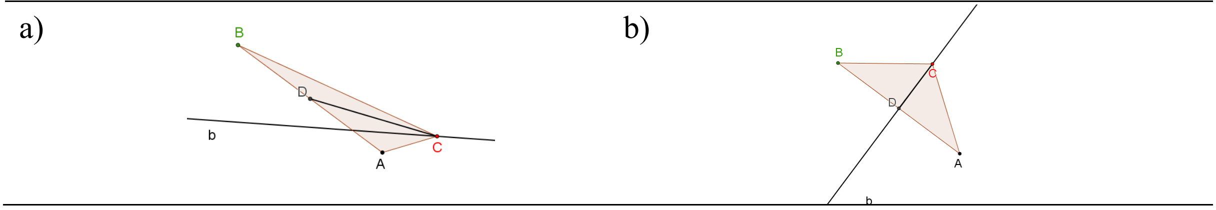

Figure 2: Standard examples of the falsifier's and the verifier's moves

The soft nature of the isosceles triangle and the game context are designed so to make the students *see* the link between the hypothesis of the theorem "the median and the angle bisector drawn from the same vertex of a triangle coincide" and its thesis "ABC is an isosceles triangle" *as* a conditional link. In fact, V's moves are guided by the intention of making the median CD and the angle bisector b coincide and the output produced by this type of move is an isosceles triangle. Once the students notice the isosceles triangles configuration, they can use it to guide the move and to provide the coincidence between median and angle bisector, reversing the conditional link. In order to foster this disclosure, questions are provided to students in a worksheet (Table 2). Q1 focuses students' attention on the geometric nature of CD and b (median and angle bisector), in the cases in which ABC is a triangle. Q2 guides students to observe the type of triangle produced by V's moves (isosceles). Finally, by exploiting the game dynamics Q3 is meant to disclose the relationships between the geometrical objects and to formulate T_{MB}.

Q1) What are CD and b with respect to the triangle ABC?
Q2) Which are the properties of the geometric figure when the verifier reaches his goal?
Q3) From the facts observed during the game and the answers given to the previous questions formulate a geometric conjecture.

Table 2: Guiding questions on the game

DATA ANALYSIS

<u>Game phase.</u> All the pairs except for one attribute the winning of all the matches to the verifier. Videorecorded students S1 and S2 follow this expected trajectory. We report

an excerpt showing their first observations after playing several matches in which the verifier has always won:

1	S2:	Impossible that the falsifier wins
2	S1:	Moving after you *(dragging C in Fig.3a)*, it's always possible to create triangles *(making Fig.3b by moving B)*. C is always on the straight-line b, hence in order to move D, which is the midpoint, it is sufficient… make b coincide… then the verifier always wins. How is it possible not to make them coincide? *(moving C on AB, see Fig.3c)*

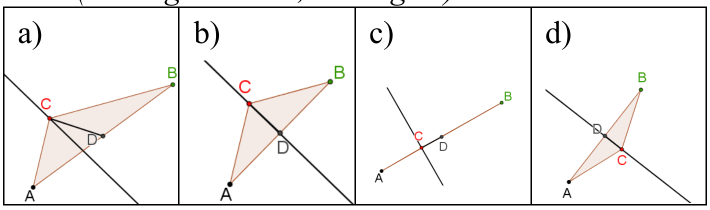

Figure 3: Configuration explored by S1 (fictitious game)

3	S2:	You can't!
4	S1	Even in this case you manage to make it *(moving B so that CD and b coincide, Fig.3d)*

The verifier's winnings are justified by S1 (who plays both the verifier's and the falsifier's role, ll. 2 and 4) through two fictitious matches: one played on a standard configuration (Fig.3a), the other played on a degenerate case (Fig.3c). These fictitious matches and their description can be interpreted within TSM as *artefact signs*. As a matter of fact, although geometric terms are used ("triangle", "midpoint"), students' attention is mainly focused on the game: they are disclosing what happens in the inquiring-game, which constitutes a first layer of meaning in the given context.

Question 1. All pairs of students easily recognize CD as the median of triangle ABC; on the contrary, they face more difficulties in discovering the geometric nature of line b, due to the confusion between the *soft properties* that line b can assume by dragging it and the *robust properties* that it always has because of its construction. Specifically, some students *see b as perpendicular to AB*: this is a *soft property* for line b, observable only in verifier's winning configuration (when the triangle is isosceles as a *soft diagram*). What is more striking is that students are disclosing a mathematical meaning for this line (a *mathematical sign* within TSM), which was *not expected* by the designers of the activity. When the teacher notices that S1 and S2 are working within this unexpected layer of meanings, he decides to intervene:

5	T:	Do you see any right angles there? *(pointing to the scalene triangle on the screen, configuration similar to Fig.3a)*
6	S2:	No, only when it [b] coincide with CD
7	T:	Ah interesting! Good point! But in general, you can't say it's perpendicular to AB, in general it has some other properties, try to understand it.
8	S1:	I don't know... I confess that I have no idea... *(starts dragging C)*

In his intervention, the teacher prompts students' attention on the line b when the triangle configuration is non-isosceles and b is not perpendicular (l. 5, configuration similar to Fig. 3a). In this way, S1 and S2 become aware of their mistake (l. 6) but they need more investigation to provide the right answer to question 1 (l. 8), which entails *seeing* line b *as something else* than a perpendicular line. A second teacher's intervention which prompts a double focus of attention ("on the property which remains true while moving the points" and "on the known elements of triangles") is necessary.

Question 2. Four pairs of students out of six give the expected answer (the triangle is isosceles). One pair writes that all points are aligned on line b. S1 and S2 write that the configuration is "always a straight-line and sometimes an isosceles triangle". This answer is a compromise between two different ways of seeing the situation: S2 is focused on the isosceles triangle, S1 is focused on the straight line that appears in the degenerate cases. The teacher decides to intervene:

9	T:	If you read question 2 here, [reads] 'which are the properties of the geometric figure when the verifier reaches his goal'... if you want, which are the properties of the triangle each time the verifier reaches the goal? That is, how would you describe that triangle?
10	S1:	Scalene
11	S2:	Isosceles
12	S1:	Why do you see it as isosceles *(looking Fig.4a)*? Maybe it is isosceles *(making Fig.4b by moving C)* Do you really see it as isosceles? *(He moves C upside and obtains Fig.4c)* Yes, ok. It's isosceles *(making Fig.4d)* So every time they coincide it is an isosceles triangle.

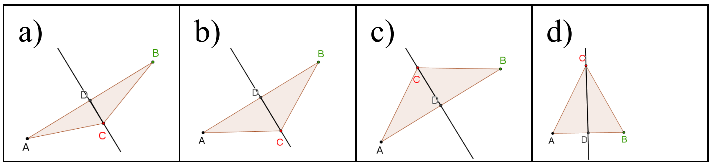

Figure 4: Getting a prototypical configuration in order to answer to question 2

The teacher reformulates the question: instead of speaking of "properties of geometric figure", he makes explicit reference to the "triangle" (l. 9). In this way, he is prompting the students' attention towards the envisaged mathematical signs involved in T_{MB}. In order to discern which kind of triangle is ABC when the verifier wins, S1 drags the points until AB is horizontal: in this prototypical configuration she is able to *see* ABC *as* an isosceles triangle (l. 12).

Question 3. Three pairs of students write the statement of the theorem using "if… then…" formulation, i.e. the mathematical sign corresponding to the expected outcome of the activity. One pair produces a pivot sign mixing artefact signs (referring to the game) with mathematical signs: "If the verifier reaches the goal then an isosceles triangle is formed". Two pairs do not get to the expected formulation of the theorem. In one case, they focus on a degenerate configuration ("If CD is the median of the triangle then aligned with the angle bisector, it makes a straight line"); in the second case, they do not mention the triangle ("If CD is perpendicular to AB then CD coincides with straight line b”). The latter answer is written by the second video-recorded pair of students. They played the game in an unexpected way, using the zoom tool to check the coincidence between b and CD (e.g. Fig. 5c) and to fix the position of point B (e.g. Fig. 5d). Since by zooming the coincidence is never accurate enough the students get lost in fixing/zooming cycles of the triangle configuration. The produced artefact signs develop a layer of meaning which cannot evolve in the expected mathematical one.

The overall data analysis shows that some of the extreme configurations produced by students in playing the game (*soft diagrams*) prompts them on unforeseen paths in which the produced artefact signs do not evolve in the expected mathematical ones. In Fig. 5 we report some of such extreme configurations, produced with the dragging tool and/or the zoom tool:

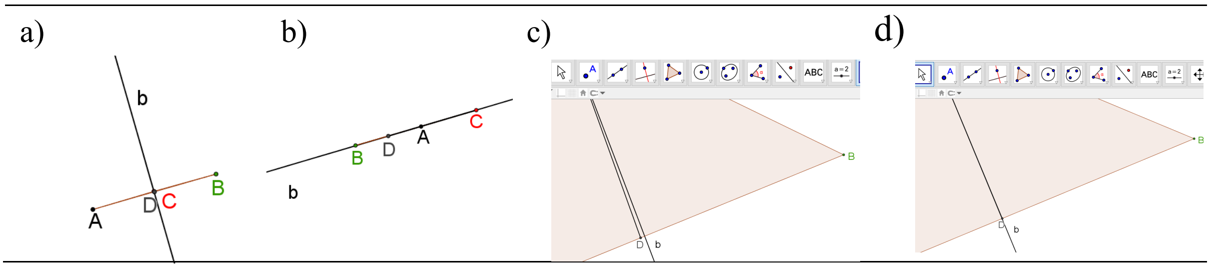

Figure 5: Extreme configurations of *soft diagrams* explored within inquiring-games

Fig.5a-b are degenerate cases in which V’s goal is reached (b and CD coincide) but the artefact signs cannot evolve in the expected mathematical sign since the triangle ABC is collapsed. Fig.5c shows a zoom-in configuration in which V’s goal is reached but the triangle ABC it is not visible because of its big dimensions.

CONCLUSION

The analysis provides evidences that through guiding questions and the teacher’s interventions most of the produced artefact/pivot/mathematical signs evoke expected layers of meanings related to the game and to the mathematical contexts, initiating students in the disclosure of the geometric theorem on which the game is based. Our hypothesis that the use of *soft configurations* allows students to experience the conditional link on which the theorem is based is confirmed (4 pairs out of 6 formulate the expected statement of the theorem as answer to question 3). Furthermore, results indicate that the game dynamics push the students in a deep exploration of both prototypical and non-prototypical situations (e.g. zoomed configurations and degenerate diagrams). The letter situations can produce unforeseen artefact signs and develop unexpected mathematical layers (e.g. seeing the angle bisector as an height; seeing the degenerate configuration as general case).

We believe that the exploration of these configurations should not be prevented to students since they can enrich their theoretical reflections and understanding of geometry. Degenerate figures can be exploited by the teacher to define the boundaries of the theorem subscribed by its hypothesis, hence they can be used to reflect deeply on the role of the hypothesis in the statement of the theorem. In our case, if ABC is not a triangle, CD cannot be the median drawn from vertex C and b cannot be the angle bisector drawn from the same vertex, since their existence depends on the triangle configuration, hence we cannot make sense of the theorem using these configurations. Our current research is investigating how, in order to discuss these situations, a class discussion on the verifier and falsifier dynamic and on the transformations produced on *soft diagrams* with the dragging tool in DGE may be managed.

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